35. $y = 2e^{-t}\cos t \quad [0, 2\pi]$

$\bar{y} = \frac{1}{2\pi - 0}\int_0^{2\pi} 2e^{-t}\cos t \, dt$

$\int 2e^{-t}\cos t \, dt = 2e^{-t}\sin t + \int 2e^{-t}\sin t \, dt$

$u = 2e^{-t} \quad du = -2e^{-t}dt$ | $u = 2e^{-t} \quad du = -2e^{-t}dt$

$dv = \cos t \, dt \quad v = \sin t$ | $dv = \sin t \, dt \quad v = -\cos t$

$\int 2e^{-t}\cos t \, dt = 2e^{-t}\sin t - 2e^{-t}\cos t - \int 2e^{-t}\cos t \, dt$

$\frac{1}{2\pi}\cancel{2}\int_0^{2\pi} 2e^{-t}\cos t \, dt = \underbrace{\cancel{2}e^{-t}\sin t - \cancel{2}e^{-t}\cos t}_{2\pi} \Big|_0^{2\pi}$

$\bar{y} = \frac{(0 - e^{-2\pi}) - (0 - e^0)}{2\pi} = \frac{1 - e^{-2\pi}}{2\pi}$

Nov 30-9:14 AM

43 $\int \sin\sqrt{x}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$2\sqrt{x}\,du = dx$

$\int \sin u \cdot 2\sqrt{x}\,du$

$\int 2u \cdot \sin u \, du = -2u\cos u + 2\sin u + C$

$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$

$2u$	$\sin u$
2	$-\cos u$
0	$-\sin u$

Nov 30-9:27 AM

6.4a Exponential Growth and Decay

law of exponential change diff eq.

y is proportional to x

$y = kx \quad k = \frac{y}{x}$

If y changes at a rate proportional to the amount present

that is, if $\frac{dy}{dt} = ky$ then $\boxed{y = y_0 e^{kt}}$

memorize

$\frac{dy}{dt} = ky$

sep of variables

$dy = ky \, dt$

$\int \frac{dy}{y} = \int k \, dt$

$\ln|y| = kt + c$

$e^{\ln|y|} = e^{kt+c}$

$|y| = e^{kt} \cdot e^c$

$y = e^{kt} \cdot A$

initial condition

$t = 0 \quad y = y_0$

$y_0 = e^0 \cdot A = A$

$y = y_0 e^{kt}$

Dec 2-1:38 PM

Suppose you deposit \$800 (y_0) in an account that pays 6.3% ($k = .063$) annual interest compounded continuously. How much do you have 8 years later?

$y = y_0 e^{kt}$

$y = 800e^{.063t}$

$y = 800e^{.063 \cdot 8}$

$y = 1324.26$

Find the doubling time.

$1600 = 800e^{.063t}$

$2 = e^{.063t}$

$\ln 2 = \ln e^{.063t}$

$\ln 2 = .063t$

$11.0023 = \frac{\ln 2}{.063} = t$

$\boxed{D = \frac{\ln 2}{k}}$

Dec 2-1:45 PM

Radioactive decay and half-life

$\frac{dy}{dt} = -ky$ $y = y_0 e^{-kt}$

$\boxed{H = \frac{\ln 2}{k}}$

Find the half-life $y = \frac{y_0}{2}$

$\frac{y_0}{2} = y_0 e^{-kt}$

$\frac{1}{2} = e^{-kt}$

$\ln\frac{1}{2} = -kt$

$t = \frac{\ln\frac{1}{2}}{-k}$

$t = \frac{\ln 1 - \ln 2}{-k}$

$\boxed{t = \frac{\ln 2}{k}}$

Dec 2-1:47 PM

Choosing a convenient base

$y = y_0 2^{\frac{t}{D}}$ D=Doubling Time

$y = y_0 (\frac{1}{2})^{\frac{t}{H}}$ H = Half-life

Dec 2-1:53 PM

The half-life of carbon 14 is about 5700 years. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

use to find k

(90% remains)

90% of y_0 remains so $y = .90\, y_0$

$k = \frac{\ln 2}{5700} = .000122$

$y = y_0 e^{-.000122t}$

$.90 \cancel{y_0} = \cancel{y_0} e^{-.000122t}$

$\ln .90 = -.000122t$

$\frac{\ln .90}{-.000122} = t$

$t = 866$

Dec 2-1:57 PM

Ex 4. At the beginning of the summer the population of a hive of wasps is growing at a rate proportional to the population. From a population of 10 on May 1, the number of hornets grows to 50 in 30 days. If the growth continues to follow the same model, how many days after May 1 will the population reach 100?

$t = 30 \quad y = 50$

$y_0 = 10$

$y = y_0 e^{kt}$

$50 = 10 e^{k \cdot 30}$

$5 = e^{k \cdot 30}$

$\ln 5 = 30k$

$k = \frac{\ln 5}{30}$

$y = 100 = 10 e^{\frac{\ln 5}{30}t}$

$10 = e^{\frac{\ln 5}{30} \cdot t}$

$\ln 10 = \frac{\ln 5}{30} t$

$t = \frac{30}{\ln 5} \cdot \ln 10$

$t = 43$

Dec 2-2:00 PM